

# Towards a SDLCQ test of the Maldacena Conjecture

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## Abstract

We consider the Maldacena conjecture applied to the near horizon geometry of a D1-brane in the supergravity approximation and present numerical results of a test of the conjecture against the boundary field theory calculation using DLCQ. We previously calculated the two-point function of the stress-energy tensor on the supergravity side; the methods of Gubser, Klebanov, Polyakov, and Witten were used. On the field theory side, we derived an explicit expression for the two-point function in terms of data that may be extracted from the supersymmetric discrete light cone quantization (SDLCQ) calculation at a given harmonic resolution. This yielded a well defined numerical algorithm for computing the two-point function. For the supersymmetric Yang-Mills theory with 16 supercharges that arises in the Maldacena conjecture, the algorithm is perfectly well defined; however, the size of the numerical computation prevented us from obtaining a numerical check of the conjecture. We now present numerical results with approximately 1000 times as many states as we previously considered. These results support the Maldacena conjecture and are within 10 – 15% of the predicted numerical results in some regions. Our results are still not sufficient to demonstrate convergence, and, therefore, cannot be considered to a numerical proof of the conjecture. We present a method for using a “flavor” symmetry to greatly reduce the size of the basis and discuss a numerical method that we use which is particularly well suited for this type of matrix element calculation.

# 1 Introduction

The discovery that certain field theories admit concrete realizations as a string theory on a particular background has caused a great deal of excitement in recent years [1]. However, attempts to apply these correspondences to study the details of these theories have only met with limited success so far. The problem stems from the fact that our understanding of both sides of the correspondence is limited. On the field theory side, most of what we know comes from perturbation theory where we assume that the coupling is weak. On the string theory side, most of what we know comes from the supergravity approximation where the curvature is small. There are no known situations where both approximations are simultaneously valid. At the present time, comparisons between the dual gauge/string theories have been restricted to either qualitative issues or quantities constrained by symmetry. Any improvement in our understanding of field theories beyond perturbation theory or string theories beyond the supergravity approximation is, therefore, a welcome development.

Previously [2] we showed that Supersymmetric Discrete Light Cone Quantization (SDLCQ) of field theories [3, 4, 5, 6] can, in principle, be used to make a quantitative comparison with the supergravity approximation on the string theory side of the correspondence. We discussed this in two space-time dimensions where the SDLCQ approach works particularly well; however, it can in principle be extended to more dimensions.

We will study the field theory/string theory correspondence motivated by considering the near-horizon decoupling limit of a D1-brane in type IIB string theory [8]. The gauge theory corresponding to this theory is the Yang-Mills theory in two dimensions with 16 supercharges. Its SDLCQ formulation was recently reported in [9], and recent work has put the use of SDLCQ for this class of problems on a stronger footing [7]. This is probably the simplest known example of a field theory/string theory correspondence involving a field theory in two dimensions with a concrete Lagrangian formulation.

A convenient quantity that can be computed on both sides of the correspondence is the correlation function of gauge invariant operators [10, 11]. We will focus on two-point functions of the stress-energy tensor. This turns out to be a very convenient quantity to compute for reasons that are discussed in [2]. Some aspects of this, as it pertains to a consideration of black hole entropy, were recently discussed in [12]. In the DLCQ literature, the spectrum of hadrons is often reported [5]. This would be fine for theories in a confining phase. However, we expect the SYM in two dimensions to flow to a non-trivial conformal fixed point in the infra-red. The spectrum of states will therefore form a continuum and will be cumbersome to handle. On the string theory side, entropy density and the quark anti-quark potential are frequently reported. The definition of entropy density requires that

we place the field theory in a space-like box which is incommensurate with the light-like box of DLCQ. Similarly, a static quark anti-quark configuration does not fit very well inside a discretized light-cone geometry. A correlation function of point-like operators does not suffer from these problems.

## 2 Correlation functions in supergravity

The correlation function of the stress-energy tensor on the string theory side, with use of the supergravity approximation, was presented in [2], and we will only quote the result here. The computation is essentially a generalization of [10, 11]. The main conclusion on the supergravity side was reported recently in [12]. Up to a numerical coefficient of order one, which we have suppressed, we found that

$$\langle \mathcal{O}(x) \mathcal{O}(0) \rangle = \frac{N_c^{\frac{3}{2}}}{g_{YM} x^5}. \quad (1)$$

This result passes the following important consistency test. The SYM in 2 dimensions with 16 supercharges have conformal fixed points in both UV and IR with central charges of order  $N_c^2$  and  $N_c$ , respectively. Therefore, we expect the two point function of the stress-energy tensor to scale like  $N_c^2/x^4$  and  $N_c/x^4$  in the deep UV and IR, respectively. According to the analysis of [8], we expect to deviate from these conformal behaviors and cross over to a regime where the supergravity calculation can be trusted. The crossover occurs at  $x = 1/g_{YM}\sqrt{N_c}$  and  $x = \sqrt{N_c}/g_{YM}$ . At these points, the  $N_c$  scaling of (1) and the conformal result match in the sense of the correspondence principle [13].

## 3 Correlation functions in SUSY with 16 Super Charges

The challenge then is to attempt to reproduce the scaling relation (1), fix the numerical coefficient, and determine the details of the crossover behavior using SDLCQ. In order to actually evaluate the correlation functions, we must resort to numerical analysis.

The technique of SDLCQ is reviewed in [4], so we will be brief here. The basic idea of light-cone quantization is to parameterize space-time using light-cone coordinates  $x^+$  and  $x^-$  and to quantize the theory making  $x^+$  play the role of time. In the discrete light cone approach, we require the momentum  $p_- = p^+$  along the  $x^-$  direction to take on discrete values in units of  $p^+/K$  where  $p^+$  is the conserved total momentum of the system and  $K$  is an integer commonly referred to as the harmonic resolution [3]. One can think of this discretization as a consequence of compactifying the  $x^-$  coordinate on a circle with

a period  $2L = 2\pi K/p^+$ . The advantage of discretizing on the light cone is the fact that the dimension of the Hilbert space becomes finite. Therefore, the Hamiltonian is a finite dimensional matrix, and its dynamics can be solved explicitly. In SDLCQ one makes the DLCQ approximation to the supercharges, and these discrete representations satisfy the supersymmetry algebra. Therefore SDLCQ enjoys the improved renormalization properties of supersymmetric theories. Of course, to recover the continuum result, we must send  $K$  to infinity and as luck would have it, we find that SDLCQ usually converges faster than the naive DLCQ. Of course, in the process the size of the matrices will grow, making the computation harder and harder.

Let us now return to the problem at hand. We would like to compute a general expression of the form  $F(x^-, x^+) = \langle \mathcal{O}(x^-, x^+) \mathcal{O}(0, 0) \rangle$ . In DLCQ, where we fix the total momentum in the  $x^-$  direction, it is more natural to compute the Fourier transform and express the transform in a spectral decomposed form

$$\tilde{F}(P_-, x^+) = \frac{1}{2L} \langle \mathcal{O}(P_-, x^+) \mathcal{O}(-P_-, 0) \rangle = \sum_i \frac{1}{2L} \langle 0 | \mathcal{O}(P_-) | i \rangle e^{-iP_+^i x^+} \langle i | \mathcal{O}(-P_-, 0) | 0 \rangle. \quad (2)$$

The position-space form of the correlation function is recovered by Fourier transforming with respect to  $P_- = K\pi/L$ . We can continue to Euclidean space by taking  $r = \sqrt{2x^+x^-}$  to be real. The result for the correlator of the stress-energy tensor was presented in [2], and we only quote the results here:

$$F(x^-, x^+) = \sum_i \left| \frac{L}{\pi} \langle 0 | T^{++}(K) | i \rangle \right|^2 \left( \frac{x^+}{x^-} \right)^2 \frac{M_i^4}{8\pi^2 K^3} K_4(M_i \sqrt{2x^+x^-}), \quad (3)$$

where  $M_i$  is a mass eigenvalue and  $K_4(x)$  is the modified Bessel function of order 4. In [9] we found that the momentum operator  $T^{++}(x)$  is given by

$$T^{++}(x) = \text{tr} \left[ (\partial_- X^I)^2 + \frac{1}{2} (iu^\alpha \partial_- u^\alpha - i(\partial_- u^\alpha) u^\alpha) \right], \quad I, \alpha = 1 \dots 8 \quad (4)$$

where  $X$  and  $u$  are the physical adjoint scalars and fermions respectively, following the notation of [9]. When discretized, these operators have the mode expansions

$$\begin{aligned} X_{i,j}^I &= \frac{1}{\sqrt{4\pi}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \left[ a_{ij}^I(n) e^{-i\pi n x^- / L} + a_{ji}^{\dagger I}(n) e^{i\pi n x^- / L} \right], \\ u_{i,j}^\alpha &= \frac{1}{\sqrt{4L}} \sum_{n=1}^{\infty} \left[ b_{ij}^\alpha(n) e^{-i\pi n x^- / L} + b_{ji}^{\dagger \alpha}(-n) e^{i\pi n x^- / L} \right]. \end{aligned} \quad (5)$$

The matrix element  $(L/\pi) \langle 0 | T^{++}(K) | i \rangle$  is independent of  $L$  and can be substituted directly to give an explicit expression for the two-point function. We see immediately that the correlator has the correct small- $r$  behavior, for in that limit, it asymptotes to

$$\left( \frac{x^-}{x^+} \right)^2 F(x^-, x^+) = \frac{N_c^2(2n_b + n_f)}{4\pi^2 r^4} \left( 1 - \frac{1}{K} \right). \quad (6)$$

On the other hand, the contribution to the correlator from strictly massless states is given by

$$\left(\frac{x^-}{x^+}\right)^2 F(x^-, x^+) = \sum_i \left| \frac{L}{\pi} \langle 0 | T^{++}(K) | i \rangle \right|_{M_i=0}^2 \frac{6}{K^3 \pi^2 r^4}. \quad (7)$$

It is important that this  $1/r^4$  behavior at large  $r$  not be confused with the  $1/r^4$  behavior that we seek at large  $r$ . First of all, there is not supposed to be any massless physical bound state in this theory, and, secondly, it has the wrong  $N_c$  dependence.

Relative to the  $1/r^4$  behavior at small  $r$ , the  $1/r^4$  behavior at large  $r$  that we expect is down by a factor of  $1/N_c$ . Since we are doing a large- $N_c$  calculation, this behavior is suppressed. We can only hope to see the transition from the  $1/r^4$  behavior at small  $r$  to the region where the correlator behaves like  $1/r^5$ .

## 4 Discrete Symmetries of the Problem.

In order to calculate the correlation function we use the expression (2). This means that after diagonalizing the Hamiltonian  $P^-$  one should evaluate the projection of each eigenfunction on the specific state  $T^{++}(-K)|0\rangle$ . The fact that we are only interested in states which have nonzero value of such projection leads to significant simplifications.

One can diagonalize any of the eight supercharges  $Q_\alpha^-$ . In the continuum limit, the result does not depend on the value of  $\alpha$  that one chooses, but in DLCQ the situation is a little more subtle. As was shown in [9], while the spectrum of  $(Q_\alpha^-)^2$  is the same for all  $\alpha$ , the wave functions depend on the choice of supercharge. This dependence is an artifact of DLCQ and should disappear in the continuum limit. We refer to [9] for the discussion of this issue. Here we will just pick one supercharge (for example,  $Q_1^-$ ). Since the state  $T^{++}(-K)|0\rangle$  is a singlet under R-symmetry acting on the “flavor” index of  $Q_\alpha^-$ , the correlator (2) does not depend on the choice of  $\alpha$  even at finite resolution.

A significant simplification occurs at this stage. Suppose there exists an operator  $S$  commuting with both  $P^-$  and  $T^{++}(-K)$  and such that  $S|0\rangle = s_0|0\rangle$ . Then the Hamiltonian and  $S$  can be diagonalized simultaneously. From now on we assume that the set of states  $|i\rangle$  is a result of such diagonalization. In this case, only states satisfying the condition  $S|i\rangle = s_0|i\rangle$  contribute to the sum in (2), and we only need to diagonalize  $P^-$  in this sector. So if one finds a large enough set of appropriate operators  $S$ , then the size of the problem can be significantly reduced. By looking at the structure of the state  $T^{++}(-K)|0\rangle$  one can conclude, given arbitrary permutations  $P$  and  $Q$  of the 8 flavor indices, that any transformation of the

form

$$\begin{aligned} a_{ij}^I(k) &\rightarrow f(I)a_{ij}^{P[I]}(k), & f(I) &= \pm 1 \\ b_{ij}^\alpha(k) &\rightarrow g(\alpha)b_{ij}^{Q[\alpha]}(k), & g(\alpha) &= \pm 1 \end{aligned} \quad (8)$$

commutes with  $T^{++}(-K)$ , and that the vacuum is an eigenstate of this transformation with eigenvalue 1. The requirement for  $P^- = (Q_1^-)^2$  to be invariant under  $S$  imposes some restrictions on the permutations. In fact, we will require that  $Q_1^-$  be invariant under  $S$ , in order to guarantee that  $P^-$  is invariant.

The form of the supercharge from [9] is

$$Q_\alpha^- = \int_0^\infty [\dots] b_\alpha^\dagger(k_3) a_I(k_1) a_I(k_2) + \dots + (\beta_I \beta_J^T - \beta_J \beta_I^T)_{\alpha\beta} [\dots] b_\beta^\dagger(k_3) a_I(k_1) a_J(k_2) + \dots \quad (9)$$

Here the  $\beta_I$  are  $8 \times 8$  real matrices satisfying  $\{\beta_I, \beta_J^T\} = 2\delta_{IJ}$ . We use a special representation for these matrices given in [14].

Let us consider the expression for  $Q_1^-$ . The first part of the supercharge (the one which does not include  $\beta$  matrices) is invariant under (8) as long as  $g(1) = 1$  and  $Q[1] = 1$ . We will consider only such transformations. In order to analyze the symmetries of the  $\beta$  terms, let us make the following observation. In the representation of  $\beta$  matrices we have chosen, the expression  $\mathcal{B}_{IJ}^\alpha = (\beta_I \beta_J^T - \beta_J \beta_I^T)_{1\alpha}$  may take only the values  $\pm 2$  or zero. Moreover, for any pair  $(I, J)$  there is at most one value of  $\alpha$  corresponding to nonzero  $\mathcal{B}$ . This fact allows us to represent  $\mathcal{B}$  in a compact form. To do so, we introduce a new object  $\mu$  defined by

$$\mu_{IJ} = \begin{cases} \alpha, & \mathcal{B}_{IJ}^\alpha = 2 \\ -\alpha, & \mathcal{B}_{IJ}^\alpha = -2 \\ 0, & \mathcal{B}_{IJ}^\alpha = 0 \text{ for all } \alpha. \end{cases} \quad (10)$$

Our choice of  $\beta$  matrices then leads to the following expression for  $\mu$ :

$$\mu = \begin{pmatrix} 0 & 5 & -7 & 2 & -6 & 3 & -4 & 8 \\ -5 & 0 & -3 & 6 & 2 & -7 & 8 & 4 \\ 7 & 3 & 0 & -8 & -4 & -5 & 6 & 2 \\ -2 & -6 & 8 & 0 & -5 & 4 & 3 & 7 \\ 6 & -2 & 4 & 5 & 0 & -8 & -7 & 3 \\ -3 & 7 & 5 & -4 & 8 & 0 & -2 & 6 \\ 4 & -8 & -6 & -3 & 7 & 2 & 0 & 5 \\ -8 & -4 & -2 & -7 & -3 & -6 & -5 & 0 \end{pmatrix}. \quad (11)$$

We are looking for a subset of transformations (8) that satisfy the conditions  $g(1) = 1$  and  $Q[1] = 1$  and leave the matrix  $\mu$  invariant. The latter property means that

$$Q[\mu_{P[I]P[J]}] = g(\mu_{IJ})f(I)f(J)\mu_{IJ}. \quad (12)$$

Since the subset of transformations that we seek forms a subgroup  $R$  of the permutation group  $S_8 \times S_8$ , it is natural to look for the elements of  $R$  that square to one. In the case of

$S_8 \times S_8$ , it is known that products of such elements generate the whole group, and, as we will show later, the same is true for  $R$ . One can construct all  $Z_2$  symmetries satisfying (12), but not all of them are independent. In particular if  $a$  and  $b$  are two such symmetries then  $aba$  is also a  $Z_2$  symmetry. By studying different possibilities we have found that there are 7 independent  $Z_2$  symmetries in the group  $R$ , and we have chosen them to be

	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
1	$a_7$	$a_3$	$a_2$	$a_6$	$a_8$	$a_4$	$a_1$	$a_5$	$b_2$	$-b_3$	$-b_4$	$-b_6$	$-b_5$	$b_8$	$b_7$
2	$a_3$	$a_6$	$a_1$	$a_5$	$a_4$	$a_2$	$a_8$	$a_7$	$-b_4$	$b_3$	$-b_2$	$-b_5$	$b_8$	$-b_7$	$b_6$
3	$a_8$	$a_7$	$a_6$	$a_5$	$a_4$	$a_3$	$a_2$	$a_1$	$-b_3$	$-b_2$	$b_4$	$-b_5$	$b_7$	$b_6$	$-b_8$
4	$a_5$	$a_4$	$a_8$	$a_2$	$a_1$	$a_7$	$a_6$	$a_3$	$-b_2$	$-b_7$	$b_8$	$b_5$	$-b_6$	$-b_3$	$b_4$
5	$a_8$	$a_3$	$a_2$	$a_7$	$a_6$	$a_5$	$a_4$	$a_1$	$-b_5$	$-b_3$	$b_7$	$-b_2$	$b_6$	$b_4$	$-b_8$
6	$a_5$	$a_8$	$a_7$	$a_6$	$a_1$	$a_4$	$a_3$	$a_2$	$-b_8$	$b_5$	$-b_4$	$b_3$	$-b_6$	$b_7$	$-b_2$
7	$a_4$	$a_6$	$a_8$	$a_1$	$a_7$	$a_2$	$a_5$	$a_3$	$-b_2$	$-b_6$	$b_5$	$b_4$	$-b_3$	$-b_7$	$b_8$

Using Mathematica we explicitly constructed all the symmetries of the type (8) satisfying (12). We found that the group of such transformations has 168 elements, and we have shown that all of them can be generated from the seven  $Z_2$  symmetries mentioned above.

In our numerical procedure we use the  $Z_2$  symmetries in the following way. Since all states relevant for the correlator are singlets under the symmetry group  $R$ , we join our states in classes and treat the whole class as a new state. For instance, the simplest nontrivial singlet looks like

$$|1\rangle = \frac{1}{8} \sum_{I=1}^8 \text{tr} \left( a^\dagger(1, I) a^\dagger(K-1, I) \right) |0\rangle. \quad (13)$$

This means that if, during the construction of the basis, we encounter the state  $a^\dagger(1, 1) a^\dagger(K-1, 1) |0\rangle$  it will be replaced by the class representative (in this case, by the state  $|1\rangle$ ). Such a procedure significantly decreases the size of the basis, while keeping all the information necessary for calculating the correlator.

## 5 Numerical Results

Our numerical results are presented in Figs. 1(a) and 1(b). Figure 1(a) is a log-log plot of  $r^4$  times the correlator versus  $r$ , so that a  $1/r^4$  behavior appears as a flat line and a  $1/r^5$  behavior gives rise to a line with slope  $-1$ . In Fig. 1(b) we plot the log-log derivative, which is computed from explicit differentiation inside the sum and amounts to a replacement of  $K_4(M_i r)$  by  $M_i K_3(M_i r)$ .

Computing this correlator beyond the small- $r$  asymptotics represents a formidable technical challenge. In [9] we were able to construct the mass matrix explicitly and compute the

spectrum for  $K = 2$ ,  $K = 3$ , and  $K = 4$ . Even for these modest values of the harmonic resolution, the Hilbert space contained thousands of states. Previously in [2] we used this spectrum and the associated wave function to calculate the correlator beyond the small- $r$  region. In the calculation we present here we have made three improvements which have allowed us to expand the space by a factor of approximately 1000. The first and most straightforward improvement was to rewrite the code in C++, which simply runs faster than the Mathematica code and can be exported to faster machines. The second was to use the discrete flavor symmetry to reduce the size of the problem at a given resolution. The third improvement is a numerical algorithm that replaces the explicit diagonalization with an efficient but accurate approximation.

This numerical algorithm follows from the observation that the contributions to the eigenstate sum are weighted by the square of the projection  $\langle i | T^{++}(-K) | 0 \rangle$ . The Lanczos diagonalization algorithm [15] will naturally generate the states with nonzero projection if  $T^{++}(-K) | 0 \rangle$  is used as the starting vector. Let  $|u_1\rangle$  be the normalized vector proportional to  $T^{++}(-K) | 0 \rangle$ , set  $b_1 = 0$ , and construct a sequence of normalized vectors  $|u_n\rangle$  according to the Lanczos iteration  $b_{n+1} |u_{n+1}\rangle = P^- |u_n\rangle - a_n |u_n\rangle - b_n |u_{n-1}\rangle$ , with  $a_n = \langle u_n | P^- | u_n \rangle$ . The  $|u_n\rangle$  form an orthonormal basis with respect to which  $P^-$  is tridiagonal and easily exponentiated. Because all of these vectors are generated by applying powers of  $P^-$  to  $|u_1\rangle$ , only those eigenvectors with nonzero projections on  $|u_1\rangle$  can appear. Although generating a complete basis by iteration can yield the exact answer,<sup>1</sup> doing many fewer iterations, even 20, can be sufficient to capture the important contributions. Such an approach to the computation of a matrix element is related to work by Haydock [16] and others [17] on matrix elements of resolvents.

Before discussing our results we need to address the question of massless states. Our SDLCQ calculation of the spectrum of the (8,8) theory saw massless states [9], and we argued that they were not normalizable bound states. The argument in that paper was not completely correct but the conclusion remains true. We find that in these massless states the number of partons in all the contributions is either all even or all odd depending on whether the resolution is even or odd.

We have not, however, removed these unphysical states from the data sets but rather used them to obtain an estimate of the region in  $r$  where the calculation breaks down. This region is where the unphysical massless states dominate the correlator sum. Unfortunately, this is also the region where we expect the true large- $r$  behavior to dominate the correlator, if only the extra states were absent. The correlator is only sensitive to the two-particle content

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<sup>1</sup>Both this statement about the complete basis and the previous statement about nonzero projections will hold only in exact arithmetic. Round-off errors will eventually destroy these relationships as the Lanczos iteration proceeds.



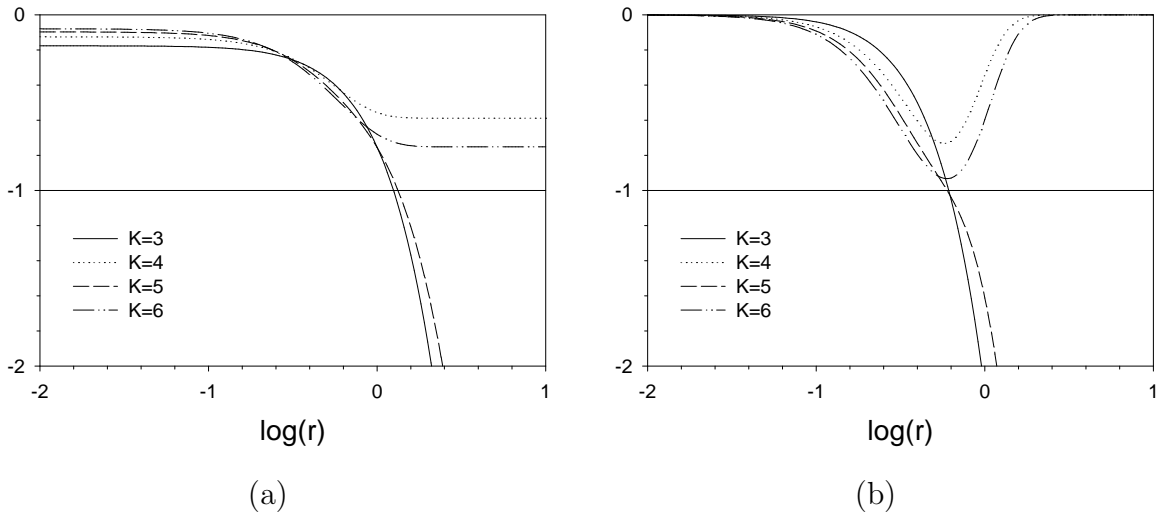


Figure 1: (a) the Log-Log plot of the correlation function  $\langle T^{++}(x)T^{++}(0) \rangle \left(\frac{x^-}{x^+}\right)^2 \frac{4\pi^2 r^4}{N_c^2(2n_b+n_f)}$  v.s.  $r$  in the units where  $g_{YM}^2 N_c / \pi = 1$  for  $K = 3, 4, 5$  and  $6$  (b) the log-log derivative with respect to  $r$  of the correlation function in (a).

of the wave function, and we see in Fig. 1(b) the characteristic behavior of the massless states at large  $r$  only at even resolutions. In Fig. 1(a) for even resolution, the region where the correlator starts to behave like  $1/r^4$  is clearly visible. In Fig. 1(b) we see that for even resolution the effect of the massless state on the derivative is felt at smaller values of  $r$  where the even resolution curves start to turn up. We use these smaller values to estimate the value of  $r$  where the large- $N_c$  approximation breaks down. We see that the value increases as we increase the resolution, as expected. Another estimate of where this approximation breaks down, that gives consistent values, is the set of points where the even and odd resolution derivative curves cross. We do not expect these curves to cross on general grounds, based on work in [2], where we considered a number of other theories.

A proof of the Maldacena conjecture would show up in Fig. 1(b) as a set of derivative curves that approached and then touched the line at  $-1$  as we increased the resolution. Convergence in the resolution,  $K$ , would appear as a flattening of the derivative curves at  $-1$  for the highest values of  $K$ .

We see that the derivative curves are approaching  $-1$  as we increase the resolution and appear to be within  $10 - 15\%$  before the approximation breaks down. There is however no indication of convergence yet; therefore, we cannot claim a numerical proof of the Maldacena conjecture.

## 6 Conclusion

In this article, we used the SDLCQ prescription for computing the correlation function of the stress-energy tensor  $T^{++}$ , which may be readily compared with predictions provided by a supergravity analysis following the conjecture of Maldacena [1]. Such a comparison requires non-perturbative methods on the field theory side, and the SDLCQ approach is the only numerical method suited to this task. At the present time the calculation gives results that are within 10 – 15% of the predicted value; however, higher resolution calculations are needed to prove convergence. The results we present here increase the number of states by a factor of 1000 relative to [2]. There are currently available methods that we believe could give us another factor of 100-1000; however, we have noted in our analysis of our numerical results that most of the contributions to the matrix element come from a very small number of eigenfunctions. An analytical understanding of this phenomenon could greatly accelerate the calculation.

Finally, we note that, in principle, we could study the proper  $1/r$  behavior at large  $r$  by computing the  $1/N_c$  corrections. In the past we have computed such corrections in some theories. However, in the present case such a computation seems to be a very large project indeed.

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## References

- [1] J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231, hep-th/9711200.
- [2] F. Antonuccio, O. Lunin, S. Pinsky, and A. Hashimoto, *JHEP* **07** (1999) 029.
- [3] H.-C. Pauli and S.J. Brodsky, *Phys. Rev.* **D32** (1985) 1993, 2001. T. Maskawa and K. Yamawaki, *Prog. Theor. Phys.* **56** (1976) 270.
- [4] S.J. Brodsky, H.-C. Pauli, and S.S. Pinsky, *Phys. Rept.* **301** (1998) 299, hep-ph/9705477.
- [5] O. Lunin and S. Pinsky, in the proceedings of 11th International Light-Cone School and Workshop: New Directions in Quantum Chromodynamics and 12th Nuclear Physics

Summer School and Symposium (NuSS 99), Seoul, Korea, 26 May - 26 Jun 1999 (New York, AIP, 1999), p. 140, hep-th/9910222.

- [6] Y. Matsumura, N. Sakai, and T. Sakai, *Phys. Rev.* **D52** (1995) 2446, hep-th/9504150.
- [7] A. Bilal, “Remarks on Defining the DLCQ of Quantum Field Theory as a Light-Like Limit,” (1999), hep-th/9909220.
- [8] N. Itzhaki, J. Maldacena, J. Sonnenschein, and S. Yankielowicz, *Phys. Rev.* **D58** (1998) 046004, hep-th/9802042.
- [9] F. Antonuccio, O. Lunin, H.-C. Pauli, S. Pinsky, and S. Tsujimaru, *Phys. Rev.* **D58** (1998) 105024, hep-th/9806133.
- [10] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, *Phys. Lett.* **B428** (1998) 105, hep-th/9802109.
- [11] E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 253, hep-th/9802150.
- [12] A. Hashimoto and N. Itzhaki, “A Comment on the Zamolodchikov  $c$  function and the black string entropy,” (1999), hep-th/9903067.
- [13] G.T. Horowitz and J. Polchinski, *Phys. Rev.* **D55** (1997) 6189, hep-th/9612146.
- [14] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory*, Vol. 1, (Cambridge, Cambridge University Press, 1987).
- [15] C. Lanczos, *J. Res. Nat. Bur. Stand.* **45**, 255 (1950); J. Cullum and R.A. Willoughby, *Lanczos Algorithms for Large Symmetric Eigenvalue Computations*, Vol. I and II, (Birkhauser, Boston, 1985).
- [16] R. Haydock, in *Solid State Physics*, Vol. 35, edited by H. Ehrenreich, F. Seitz, and D. Turnbull, (Academic, New York, 1980), p. 283.
- [17] W.H. Miller and B.M.D.D. Jansen op de Haar, *J. Chem. Phys.* **86**, 6213 (1987); C. Duneczky and R.E. Wyatt, *J. Chem. Phys.* **89**, 1448 (1988); C. Duneczky and R.E. Wyatt, *J. Phys.* **B21**, 3727 (1988); C.M.M. Nex, *Comput. Phys. Commun.* **53**, 141 (1989); C. Duneczky *et al.*, *Comput. Phys. Commun.* **53**, 357 (1989); W. Yang and W.H. Miller, *J. Chem. Phys.* **91**, 3504 (1989); H.D. Meyer and S. Pal, *J. Chem. Phys.* **91**, 6195 (1989); H.D. Meyer, J. Horáček, and L.S. Cederbaum, *Phys. Rev.* **A43**, 3587 (1991); J.R. Hiller, *Phys. Rev.* **D43**, 2418 (1991); T.J. Godin and R. Haydock, *Comput. Phys. Commun.* **64**, 123 (1991).